UNIT–I

1. a) If $A=\{1,4\}$, $B=\{4,5\}$, $C=\{5,7\}$, determine

(i). $(A\times B)\cup(A\times C)$

(ii). $(A\times B)\cap(A\times C)$

Solution: Give: $A=\{1,4\}$, $B=\{4,5\}$, $C=\{5,7\}$

Now, $A\times B=\{(1,4),(1,5),(4,4),(4,5)\}$ and $A\times C=\{(1,5),(1,7),(4,5),(4,7)\}$

$\therefore$ $(A\times B)\cup(A\times C)=\{(1,4),(1,5),(4,4),(4,5),(1,7),(4,7)\}$ **Answer**

and $(A\times B)\cap(A\times C)=\{(1,5),(4,5)\}$ **Answer**

b) Let $A=\{2,3,4\}$ and $B=\{3,4,5,6,7\}$. Assume a relation R from A to B such that (x, y)∈R when a divides 6.

Solution: Given: $A=\{2,3,4\}$ and $B=\{3,4,5,6,7\}$

Since R is a relation from A to B such that (x, y)∈R, when a divides 6 i.e. x and y both divisor of 6.

$\therefore$ $R=\{(2,3),(3,3)\}$ **Answer**

c) Briefly explain the application of Pigeon hole principle using an example.

Solution: If the number of pigeon is more than the number of pigeonholes, then some pigeonhole must be occupied by two or more than two pigeons. This statement is called the Pigeon hole principle, it is also called Dirchlet Drawer Principle. This statement is also written as

"***If n pigeonholes are occupied by n + 1 or more pigeons, then at least one pigeonhole is occupied bby more than one pigeon***".

Example 1 Among 13 people there are two who have their birthdays in the same month.

Example 2 A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruit that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?

Answer: 8 + 6 + 9 − 3 + 1 = 21.

d) Show by mathematical induction:

$$1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$

Solution: Suppose: $P(n)=1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$ … (1)

Case 1: For n = 1, we get

$$P(1)=\frac{1.3.1}{3}=1$$

There statement true for n = 1 i.e. P(1) is true.

Case 2: Suppose the statement true of n = k such that

$$P(k)=1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{k(2k+1)(2k-1)}{3} \qquad \ldots (2)$$

Next we will prove the statement true for $n = k + 1$.

Now, $\left[1^2+3^2+5^2+\ldots+(2k-1)^2\right]+\left[2(k+1)-1\right]^2=P(k)+(2k+1)^2$

$$=\frac{k(2k+1)(2k-1)}{3}+(2k+1)^2 \qquad \text{from (2)}$$

$$\Rightarrow \quad =\frac{(2k+1)}{3}\left[k(2k-1)+3(2k+1)\right]=\frac{(2k+1)}{3}(2k^2-k+6k+3)$$

$$\Rightarrow \quad =\frac{(2k+1)}{3}\left[2k^2+5k+3\right]=\frac{(2k+1)}{3}\left[(k+1)(2k+3)\right]$$

$$\Rightarrow \quad =\frac{(k+1)\left[2(k+1)+1\right]\left[2(k+1)-1\right]}{3}=P(k+1)$$

Which is true for $n = k + 1$. Thus $P(k + 1)$ is true.

Hence the statement true for each positive integral values of n. Hence proved

OR

d) Let $f: R\rightarrow R$ be defined by

$$f(x)=\begin{cases}2x+1;\ x\le 0\\ x^2+1;\ x>0\end{cases}$$

Let $g: R\rightarrow R$ be defined by

$$g(x)=\begin{cases}3x-7;\ x\le 0\\ x^3 \quad ;\ x>0\end{cases}$$

Then find the composition gof.

Solution: Given, $f(x)=\begin{cases}2x+1;\ x\le 0\\ x^2+1;\ x>0\end{cases}$

Putting x = … –1, 0, 1, 2 …, we get

$f(-1)=2(-1)+1=-1$, $f(0)=2(0)+1=1$, $f(1)=1^2+1=2$ and $f(2)=2^2+1=5\ldots.$

and $g(x)=\begin{cases}3x-7;\ x\le 0\\ x^3 \quad ;\ x>0\end{cases}$

Putting x = … –1, 0, 1, 2 …, we get

$g(-1)=3(-1)-7=-10$, $g(0)=3(0)-7=-7$, $g(1)=1^3=1$ and $g(2)=2^3=8$

Since, $gof(x)=g[f(x)]$

Putting x = … –1, 0, 1, 2 …, we get

$\therefore \quad gof(-1)=g[f(-1)]=g(-1)=-10$

$gof(0)=g[f(0)]=g(1)=1$

$gof(1)=g[f(1)]=g(2)=8$

$gof(2)=g[f(2)]=g(5)=5^3=125\ldots.$

2. a) Define semi group. Write its properties.

Solution: Suppose G be any non empty set, which is defined on binary operation ∗, then (G, ∗) is said to be semi group if they satisfy the following properties

***G*₁: Closure**

If $a\in G$ and $b\in G$, then $a * b\in G$, $\forall$ $a, b\in G$

G_2 Associative

If $a, b, c \in G$, then $(a * b) * c = a * (b * c)$, $\forall$ $a, b, c\in G$.

Example: (N, •) is semi group.

b) Write short note:

(i). Monoid **(ii). Normal Subgroup**

Solution: (i). **Monoid:**

Suppose G be any non empty set, which is defined on binary operation ∗, then (G, ∗) is said to be monoid if they satisfy the following properties

G_1: Closure

If $a\in G$ and $b\in G$, then $a * b\in G$, $\forall$ $a, b\in G$

G_2 Associative

If $a, b, c \in G$, then $(a * b) * c = a * (b * c)$, $\forall$ $a, b, c\in G$.

G3 Existence of identity

If a ∈ G, then there exist e∈G such that $a * e = e * a = a$

Example: (I, •) is monoid group.

(ii). Normal Subgroup

A subgroup H of a group G is said to be a normal subgroup of G, if for every x∈G and for every h∈H, so that $x\,h\,x^{-1}\in H$

In other words, If H is normal subgroup of G, if and only if $xHx^{-1}\subseteq H$, $\forall$ $x\in G$

c) Prove that every subgroup of a cyclic group G is cyclic.

Solution: Suppose $G = \{a\}$ is a cyclic group generated by a. If $H = G$ or $\{e\}$, then obviously H is cyclic. So let H be a proper subgroup of G. The elements of H are integral power of a. if $a^n\in H$, then the inverse of a^n i.e.

$a^{-n} \in H$

$\therefore$ H contains elements which are positive as well as negative integral power of a.

Suppose m be the least positive integer such that

$a^m \in H$

then we shall prove that

$H = \{a^m\}$ i.e. H is cyclic and is generated by a^m.

Suppose a^t be any arbitrary element of H, then division algorithm, $\exists$ integer q and r, such that

$$t = mq + r,\ 0 \le r < m \qquad \ldots(1)$$

Now, $a^m \in H \Rightarrow (a^m)^q \in \mathrm{H}$

$\Rightarrow a^{mq} \in \mathrm{H}$

$\Rightarrow (a^{mq})^{-1} \in H$

$\Rightarrow a^{-mq} \in \mathrm{H}$

Also, $a^t \in H$ and $a^{-mq} \in \mathrm{H} \Rightarrow a^t \cdot a^{-mq} \in H$

$\Rightarrow a^{t-mq} \in \mathrm{H}$

$\Rightarrow a^r \in H$ [From (1)]

Now m is the least positive integer, such that

$a^m \in H,\ 0 \le r < m$

Thus r must be equal to 0, then $t = mq$ so that $a^{mq} = (a^m)^q$

Therefore, H is cyclic and a^m is a generate of H.

d) Prove that the G = {0, 1, 2, 3, 4, 5} is a finite abelian group of order 6 with respect to addition modulo 6.

Solution: The composite table under addition modulo 6.

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

G_1 **Closure Property:**

Since all the elements of composite table belongs to set G, then G is closed with respect to addition modulo 6.

G_2 **Associative Law:**

If $a, b, c \in \mathrm{G}$, then

$$a +_6 (b +_6 c) = (a +_6 b) +_6 c,\ \forall a, b, c \in \mathrm{G}$$

Example: If a = 1, b = 2 and c = 3, then

$1 +_6 (2 +_6 3) = 1 +_6 5 = 0$ and $(1 +_6 2) +_6 3 = 3 +_6 3 = 0$, then

$1 +_6 (2 +_6 3) = (1 +_6 2) +_6 3$

***G*₃** **Existence of identity:**

Suppose a$\in$G be any element, then from composite table

$a +_6 0 = 0 +_6 a = a$, $\forall a \in G$

$\therefore$ $e = 0 \in$G is the additive identity element.

***G*₄** **Existence of inverse:**

Since if a$\in$G, then $\exists$ b$\in$G such that $a +_6 b = e = 0$

From the composite table we can see that,

$0 +_6 0 = 0 \Rightarrow b = 0$, additive inverse of 0

$1 +_6 5 = 0 \Rightarrow b = 5$, additive inverse of 1

$2 +_6 4 = 0 \Rightarrow b = 4$, additive inverse of 2

$3 +_6 3 = 0 \Rightarrow b = 3$, additive inverse of 3

$4 +_6 2 = 0 \Rightarrow b = 2$, additive inverse of 4

$5 +_6 1 = 0 \Rightarrow b = 1$, additive inverse of 5

***G*₅** **Commutative identity:**

If $a \in$G, then $a +_6 b = b +_6 a$, $\forall a, b \in$G

Since G has finite number of elements, then $(G, +_6)$ is an abelain group.

OR

d) Let (*R*, +, ×) be a ring, the operation ⊗ is defined by *a* ⊗ *b* = *a* × *b* + *b* × *a*, show that (*R*, +, ×) is a commutative ring.

Solution: Statement of above question is wrong.

3. a) Prove by truth table that the following is tautology.
$(p \leftrightarrow q \wedge r) \Rightarrow (\sim r \rightarrow \sim p)$

Solution: Given statement can be written as

$$[p \leftrightarrow (q \wedge r)\ ] \Rightarrow [(\sim r) \rightarrow (\sim p)]$$

Suppose $A \equiv p \leftrightarrow (q \wedge r)$ and $B \equiv (\sim r) \rightarrow (\sim p)$
then $A \Rightarrow B$ is a tautology.
Truth table:

p	q	r	$q \wedge r$	$A \equiv p \leftrightarrow (q \wedge r)$	$\sim r$	$\sim p$	$B \equiv (\sim r) \rightarrow (\sim p)$	$A \Rightarrow B$
T	T	T	T	**T**	F	F	**T**	**T**
T	T	F	F	**F**	T	F	**F**	**T**
T	F	T	F	**F**	F	F	**T**	**T**
T	F	F	F	**F**	T	F	**F**	**T**
F	T	T	T	**F**	F	T	**T**	**T**
F	T	F	F	**T**	T	T	**T**	**T**
F	F	T	F	**T**	F	T	**T**	**T**
F	F	F	F	**T**	T	T	**T**	**T**

Thus the given statement is a tautology.

b) Obtain the principal disjunctive normal form of the following formula;-
$\sim (p \vee q) \leftrightarrow (p \wedge q)$

Solution: Given: $\sim(p \vee q) \Leftrightarrow (p \wedge q)$

$\Leftrightarrow \quad [\sim(p \vee q) \Rightarrow (p \wedge q)] \wedge [(p \wedge q) \Rightarrow \sim(p \vee q)]$

$\Leftrightarrow \quad [\sim(p \vee q) \wedge (p \wedge q)] \vee [(p \vee q) \wedge \sim(p \wedge q)] \wedge [(p \wedge q) \wedge \sim(p \vee q)] \vee [(p \vee q) \wedge \sim(p \wedge q)]$

$\Leftrightarrow \quad [\sim p \wedge \sim q \wedge p \wedge q] \vee [(p \vee q) \wedge (\sim p \vee \sim q)] \wedge [(p \wedge q) \wedge (\sim p \wedge \sim q)] \vee [(p \vee q) \wedge (\sim p \vee \sim q)]$

$\Leftrightarrow \quad [\sim p \wedge \sim q \wedge p \wedge q] \vee [(p \vee q) \wedge (\sim p \vee \sim q)]$ [By De-Margon]

$\Leftrightarrow \quad [\sim p \wedge \sim q \wedge p \wedge q] \vee [\{(p \vee q) \wedge \sim p\} \vee \{(p \vee q) \wedge \sim q\}]$ [By Distributive Law]

$\Leftrightarrow \quad [\sim p \wedge \sim q \wedge p \wedge q] \vee [(p \wedge \sim p) \vee (q \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim q)]$ [By Distributive Law]

This is required principle disjunctive normal form.

c) Investigate the validity of the following argument

$$\begin{array}{c} p \rightarrow r \\ \sim p \rightarrow q \\ q \rightarrow s \\ \hline \therefore\ \sim r \rightarrow s \end{array}$$

Solution: Here the three premises are $p \to r$, $\sim p \to q$, $q \to s$ and conclusion is $(\sim r \to s)$. The given argument will be valid if $[(p \to r) \wedge (\sim p \to q) \wedge (q \to s)] \Rightarrow (\sim r \to s)$ is a tautology.
Suppose $A \equiv \left[(p \to r) \wedge (\sim p \to q)\right] \wedge (q \to s)$ and $B = \sim r \to s$, then $A \Rightarrow B$ is a tautology.
Truth table

p	q	r	s	$p \to r$	$\sim p$	$\sim p \to q$	$q \to s$	$(p \to r) \wedge (\sim p \to q)$	A	$\sim r$	$B = \sim r \to s$	$A \Rightarrow B$
T	T	T	T	T	F	T	T	T	T	F	T	T
T	T	T	F	T	F	T	F	T	F	F	T	T
T	T	F	T	F	F	T	T	F	F	T	T	T
T	T	F	F	F	F	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T	T	T	F	T	T
T	F	T	F	T	F	T	T	T	T	F	T	T
T	F	F	T	F	F	T	T	F	F	T	T	T
T	F	F	F	F	F	T	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T	T	F	T	T
F	T	T	F	T	T	T	F	T	F	F	T	T
F	T	F	T	T	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T	F	F	F	T	T
F	F	T	F	T	T	F	T	F	F	F	T	T
F	F	F	T	T	T	F	T	F	F	T	T	T
F	F	F	F	T	T	F	T	F	F	T	F	T

Since all entries in the last column are of "T" only, therefore $A \Rightarrow B$ is a tautology. Hence the given argument is valid.

d) Design DFA and NDFA accepting all string over {0, 1}, which end in 0 but do not contain 11 as substring.

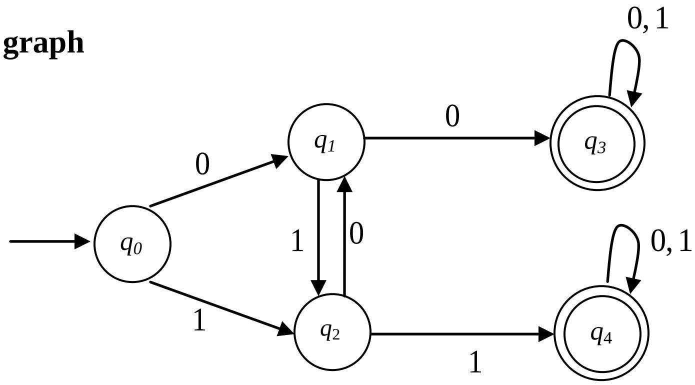
Solution: **(i) DFA (Deterministic Finite Automata)**:
The required DFA is defined by

$$M = \left[\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{q_3, q_4\}\right]$$

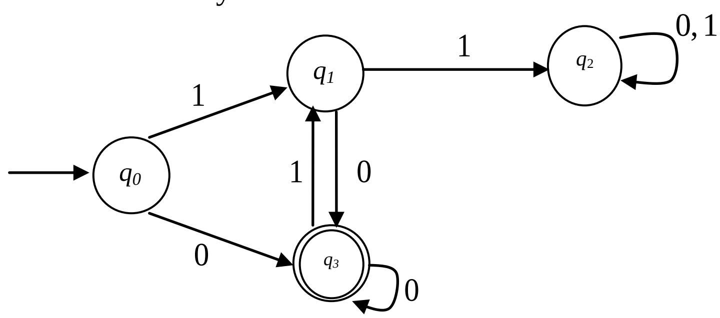
Where δ is given by

Q \ Σ	0	1
q_0	q_1	q_2
q_1	q_3	q_2
q_2	q_1	q_4
q_3	q_3	q_3
q_4	q_4	q_4

Transition graph



(i) NDFA (Non-Deterministic Finite Automata):
The required NDFA is defined by



OR

d) Prove that the validity of the following argument:
"If Ram is selected in IAS examination, then he will not be able to go to London. Since Ram is going to London, he will not be selected in IAS examination.

Solution: Suppose $p \equiv$ Ram is selected in IAS examination
$q \equiv$ Ram is going to London

The given argument, in symbolic form, may be written as

$p \rightarrow \sim q$ (Premises)
q (Premises)

$\therefore$ $\sim p$ (Conclusion)

Here two premises are $p \rightarrow \sim q$, q and conclusion is $\sim p$. The given argument will be valid if $[(p \rightarrow \sim q) \wedge (q)] \Rightarrow (\sim p)$ is a tautology.

Suppose $A \equiv (p \rightarrow \sim q) \wedge (q)$ and $B \equiv \sim p$, then $A \Rightarrow B$ is a tautology.

Truth table:

p	q	$\sim q$	$p \rightarrow \sim q$	$A \equiv (p \rightarrow \sim q) \wedge q$	$B \equiv \sim p$	$A \Rightarrow B$

T	T	F	F	F	F	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

Since all entries in the last column are of "*T*" only, therefore $A \Rightarrow B$ is a tautology. Hence the given argument is valid.

4. a) Prove that, in a graph total number of odd degree vertices is even but then number of even degree vertices may be odd.

Solution: Suppose $G = (V, E)$ be any graph in which

$V = \{v_1, v_2, v_3, \dots v_n\}$ and $E = \{e_1, e_2, e_3, \dots e_m\}$Where $n = m$ or $n \neq m$.

By Handshaking theorem we have,

The sum of degree of all the vertices of the graph, is twice the number of edges.

Therefore, $$\sum_{i=1}^{i=n} d(v_i) = 2e_m$$

$$\Rightarrow \sum_{i=odd} d(v_i) + \sum_{i=even} d(v_i) = 2m$$

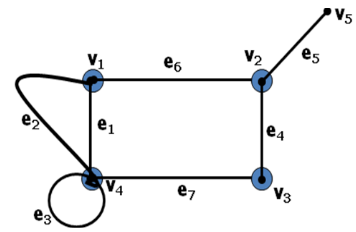
$$\Rightarrow \sum_{i=odd} d(v_i) = 2m - \sum_{i=even} d(v_i)$$

$$\Rightarrow \sum_{i=odd} d(v_i) = 2m - even\deg ree = Even$$

Thus the total number of odd degree vertices is even.

Example:

In the graph the odd degree vertices are v_1, v_2, v_4 and v_5, then
$d(v_1) + d(v_2) + d(v_4) + d(v_5) = 3 + 3 + 5 + 1 = 12$(Even)
But even degree vertex v_3 of degree is odd.

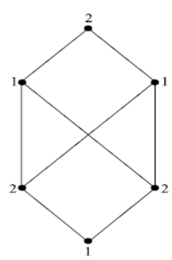


b) Distinguish between k-coloring of a graph and chromatic number of a graph.

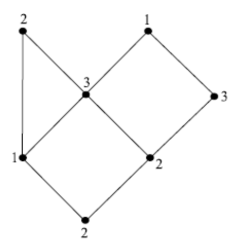
Solution: **(i). *k*–coloring:**

A vertex colouring of a graph G is a labeling $f : V(G) \rightarrow \{1, 2, \ldots\}$; the labels called colours, such that no two adjacent vertices get the same colour and each vertex gets one colour. A *k*–colouring of a graph G consists of *k* different colours and G is then called *k*-colourable.

Example:



2-colourable graph



3-colourable graph

(ii). Chromatic number:

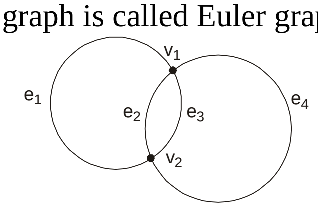
The minimum number k for which there is a *k*–colouring of the graph is called the chromatic number of G and is denoted by $\chi(G)$. If $\chi(G) = k$, the graph G is said to be *k*–chromatic.

c) Define Euler and Hamiltonian graph with example.

Solution: (i). Euler Graph:

If some closed walk in a graph *G contains all the edges* of the graph G_1, then a walk is called an **Euler line** and the graph is called an **Euler graph.**

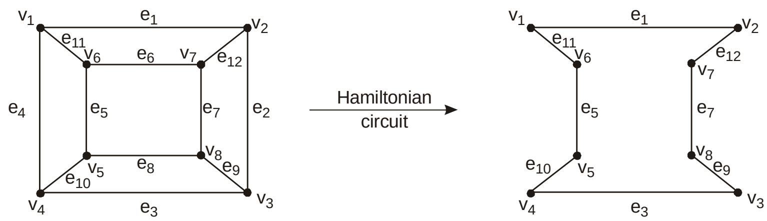
In other words, If we are moving on a graph, by covering all the edges of the graph and returning to the initial vertex, such a walk is called Euler line and graph is called Euler graph.



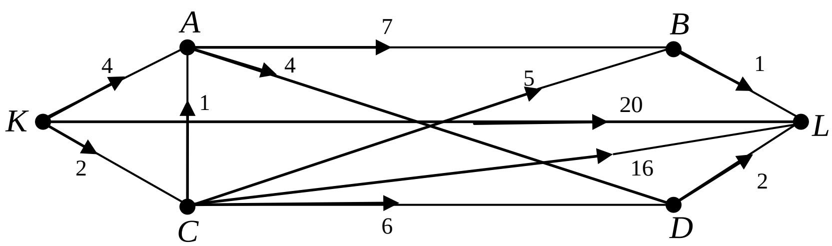
If we starting from v_1 and covering all the edges such that $v_1\ e_1\ v_2\ e_2\ v_1\ e_4\ v_2\ e_3\ v_1$.

(ii). Hamiltonian Graph:

If a closed walk contains every vertex of the graph G, such that the degree of every vertex is 2, then the walk is called Hamiltonian circuit or graph, and if the walk is open then it is said to be Hamiltonian path.



d) Find minimum distance between two vertices *K* and *L* of graph, using Disjkstra's algorithm.



Solution: Suppose

Step 1: $P_1 = \{K\}$ and $T_1 = \{A, B, C, D, L\}$

Then, we have

$l(A) = 4$, $l(C) = \mathbf{2}$ **(Minimum)** $l(B) = \infty$, $l(D) = \infty$ and $l(L) = \infty$

$\therefore$ 1 is the minimum value, then next vertex is ***C***.

Step 2: Taking, $P_2 = \{K, C\}$ and $T_2 = \{A, B, D, L\}$

Now, $l(A) = \text{minimum}(4, 2+1) = \mathbf{3}$ **(Minimum)**

$l(B) = \text{minimum}(4+7, 3+7) = 10$

$l(D) = \text{minimum}(2+6, 4+\infty) = 8$

$l(L) = \text{minimum}(20, 2+\infty, 4+\infty) = 20$

$\therefore$ 3 is minimum value, and then we take next vertex ***A***.

Step 3 Taking, $P_3 = \{K, C, A\}$ and $T_3 = \{B, D, L\}$

Now, $l(B) = \text{minimum}(3+7, 4+7, 2+5) = 7$

$l(D) = \text{minimum}(3+4, 2+6) = 7$ **(Minimum)**

$l(L) = \text{minimum}(20, 3+\infty) = 20$

$\therefore$ 7 is minimum value, and then we take next vertex ***D***.

Step 4 Taking, $P_4 = \{K, C, A, D\}$ and $T_4 = \{B, L\}$

Now, $l(B) = \text{minimum}(8+\infty, 4+7, 2+5) = 7$ **(Minimum)**

$l(L) = \text{minimum}(20, 7+2, 2+5+1) = 8$

$\therefore$ 7 is minimum value, and then we take next vertex ***B***.

Step 5 Taking, $P_5 = \{K, C, A, D, B\}$ and $T_5 = \{L\}$

Now, $l(L) = \text{minimum}(7+2, 7+1, 8+2) = 8$

Clearly the minimum path is *K, C, B, L* and length of path is 8.

OR

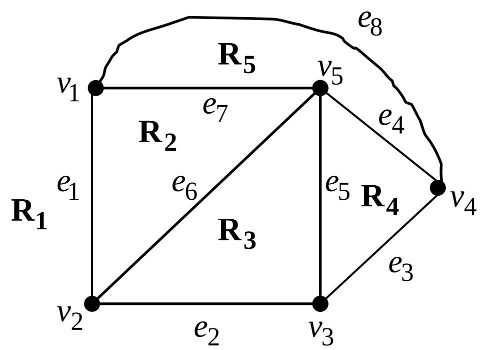
d) State Euler's formula for a planar graph. Give an example of a planar graph with 5 vertices and 5 regions and verify Euler's formula for your example.

Solution: Statement of Euler Formula:

Suppose G be a finite connected planer graph and *v* is the number of vertices, *e* is the number of edges and *f* is the number of faces (regions bounded by edges, including the outer, infinitely large region), then

$$v - e + f = 2 \qquad \dots (1)$$

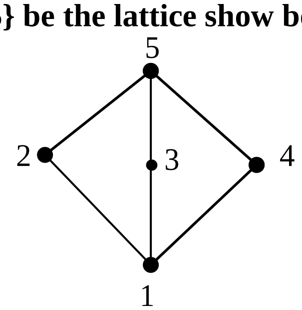
Example of 5 vertices and 5 regions of planer graph:



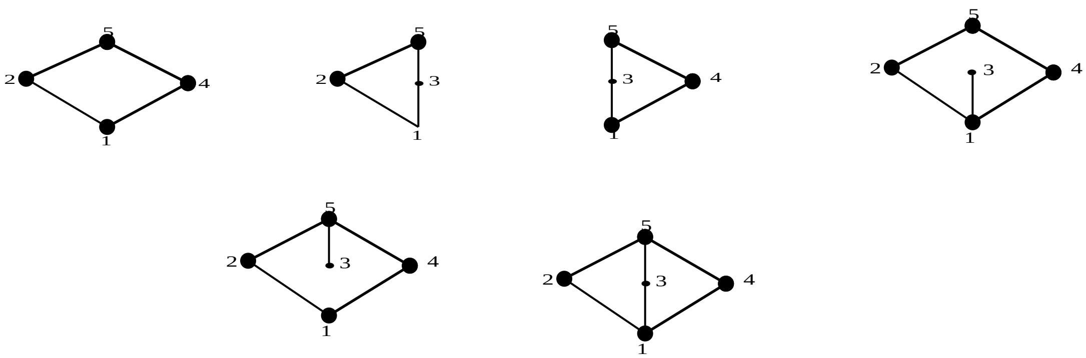
Here number of vertices *v* = 5, Number of edges e = 8 and number of Region *f* = 5

Then by Euler formula, $v - e + f = 5 - 8 + 5 = 2$

5. a) Let *L* = {1, 2, 3, 4, 5} be the lattice show below. Find all sub lattices with three or more elements.



Solution: Given the Lattice $L = \{1, 2, 3, 4, 5\}$. The sub-lattice are $S_1 = \{1, 2, 3, 4\}$, $S_2 = \{1, 2, 3, 5\}$, $S_3 = \{1, 2, 3, 4, 5\}$, $S_4 = \{1, 2, 3, 4, 5\}$ and $S_3 = \{1, 3, 4, 5\}$



b) Write down the binomial theorem.

Solution: Suppose x, y be any real value and any integer $n \geq 0$, then binomial theorem is

$$(x+y)^n = x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + {}^nC_3 x^{n-3} y^3 + \ldots + {}^nC_r x^{n-r} y^r + \ldots + y^n$$

$$\Rightarrow \qquad (x+y)^n = \sum_{r=1}^{n} {}^nC_r x^{n-r} y^r$$

c) Draw hasse diagram for the "less than or equal to" relation on set A = {0, 2, 5, 10, 11, 15}

Solution: Given the set A = {0, 2, 5, 10, 11, 15} and R be a relation of less than or equal to, then
R = {(0, 2), (2, 5), (5, 10), (10, 11), (11, 15)}
The required Hasse Diagram is
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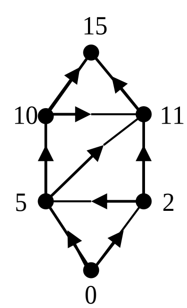

d) Determine the particular solution of the recurrence relation

$$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$$

Solution: Given, $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ …. (1)

For homogeneous solution, the characteristic equation is

$$m^2 + 5m + 6 = 0$$

$\Rightarrow$ $(m+2)(m+3)=0$

$\Rightarrow$ $m=-2,-3$

$\therefore$ Homogeneous solution is

$$a_r^{(h)}=c_1(-2)^r+c_2(-3)^r \quad \ldots. (2)$$

For the particular solution $f(r)=r^2$ is a polynomial of degree 2, so that P.S. is

$$a_r=A_0+A_1r+A_2r^2 \quad \ldots. (3)$$

Putting in equation (1), we get

$$\left[A_0+A_1r+A_2r^2\right]+5\left[A_0+A_1(r-1)+A_2(r-1)^2\right]+6\left[A_0+A_1(r-2)+A_2(r-2)^2\right]=r^2$$

$\Rightarrow$ $$\left[A_0+A_1r+A_2r^2\right]-5\left[A_0+A_1r-A_1+A_2r^2+A_2-2A_2r\right]+6\left[A_0+A_1r-2A_1+A_2r^2+4A_2-4A_2r\right]=r^2$$

$\Rightarrow$ $$12A_2r^2+r(12A_1-34A_2)+(12A_0-17A_1+29A_2)=r^2$$

Equating the coefficient of r^2, r and constant terms, we get

$$12A_2=1 \quad \Rightarrow \quad A_2=\frac{1}{12}$$

and $$12A_1-34A_2=0 \quad A_1=\frac{34}{12}A_2=\frac{17}{6}\left(\frac{1}{12}\right)=\frac{17}{72}$$

$\Rightarrow$ $$12A_0-17A_1+29A_2=0$$

$\Rightarrow$ $$A_0=\frac{17}{12}A_1-\frac{29}{12}A_2=\frac{17}{12}\left(\frac{17}{72}\right)-\frac{29}{12}\left(\frac{1}{12}\right)=\frac{289}{864}-\frac{29}{144}=\frac{115}{864}$$

Putting in equation (3), we get

$$a_r^{(p)}=\frac{115}{864}+\frac{17}{72}r+\frac{1}{12}r^2$$

The total solution of equation (1) is,

$$a_r=a_r^{(h)}+a_r^{(p)}$$

$\Rightarrow$ $$\boxed{a_r=c_1(-2)^r+c_2(-3)^r+\frac{115}{864}+\frac{17}{72}r+\frac{1}{12}r^2}$$ **Answer**

OR

d) Explain briefly:-

(i). Poset (ii). Permutation

(ii). Combination (iv) Total solutions

Solution: **(i). Poset**

A set on which a partial ordering relation is defined, is called a partially ordered set or poset.

Example: Suppose I^+ be the set of positive integers, then usual notation ≤ (Less than or equal to) is a poset on I^+, So, $(I^+, \leq)$ is a poset.

(ii). Permutation:

The different arrangements which can be made out of a given number of objects by taking some or all it a time, are called permutations.

Suppose n objects, can be arranged in r ways, then it is denoted by ${}^nP_r$ or $P(n, r)$ and it defined as

$$^nP_r = \frac{\lfloor n}{\lfloor n-r} = n(n-1)(n-2)....\left(n-\overline{r-1}\right)$$

Example: The number of permutations formed by the letters of ELORA is ${}^5P_2 = \frac{\lfloor 5}{\lfloor 5-2} = \frac{5\times4\times\lfloor 3}{\lfloor 3} = 20$

(iii). Combination:

Each of the different groups or selections which can be obtained by taking r objects of n given objects, irrespective of their arrangements, is called a combination.

Suppose n objects in which r objects to be selected, then it is denoted by ${}^nC_r$ or $C(n, r)$ and

It is defined as

$$^nC_r = \frac{\lfloor n}{\lfloor r\,\lfloor n-r} = \frac{n(n-1)(n-2)...(n-\overline{r-1})}{r.(r-1).(r-2)....3.2.1}$$

Example: The number of combinations can be formed by taking 2 letters at a time from the letters of DELHI is

$$^5C_2 = \frac{\lfloor 5}{\lfloor 5-2 \lfloor 2} = \frac{5\times 4\times \lfloor 3}{2\times 1(\lfloor 3)} = 10$$

(iv). Total solutions

The solution of any difference equation is called the total solution.

Suppose we have any difference equation, then we will two solution one is called Auxiliary solution say $a_r^{(h)}$ and another is called particular solution say $a_r^{(p)}$, then total solution of difference equation is

$$a_r = a_r^{(h)} + a_r^{(p)}$$