Std. 12 Time : 3 hrs.

17-9-2016 Half Yearly Examination in **MATHEMATICS (Set - 1)** M. Marks :100

GENERAL INSTRUCTIONS:

i) Attempt all the questions.

ii) Section - A consists of 4 questions of 1 mark each.

iii) Section-B consists of 8 questions of 2 marks each.

iv) Section- C consists of 11 questions of 4 marks each.

v) Section- D consists of 6 questions of 6 mark each.

SECTION - A

1. Evaluate: $\tan^{-1} 1 + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

2. If A $=\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ find A^4.

3. If A = $[a_{ij}]$ is a 2×2 matrix such that $a_{ij} = i^2 - j^2$, write A.

4. If A is a square matrix of order 3 and $|2A| = k\,|A|$. Find the value of k.

SECTION - B

5. Evaluate: $\int \frac{\sin x}{\sin(x+a)}\,dx$.

6. Prove that the diagonal elements of a skew symmetric matrix are all zeroes.

7. Write $\tan^{-1}\frac{\cos x}{1+\sin x}$ in the simplest form where $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$.

8. If A$=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 5A + 7I = 0$, then find A^{-1} using the given equation.

9. If $\sqrt{x} + \sqrt{y} = 1$, prove that $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is -1.

10. Evaluate: $\sin(3\sin^{-1} 0.4)$.

11. In a departmental store, a customer X purchases 2 packets of tea, 4 kg of rice and 5 dozen oranges. Customer Y purchases 1 packet of tea, 5 kg of rice and 24 oranges. Price of one packet of tea is Rs. 54, 1kg of rice is Rs. 22 and that of 1 dozen of oranges is Rs. 27. Use matrix method to solve each individual bill.

12. If $y = e^{m\sin^{-1}x}$, then show that $(1- x^2)\,y_2 - xy_1 - m^2y = 0$.

SECTION - C

13. Evaluate: $\int \frac{2x^2}{(x^2-1)(x^2+4)}$dx.

14. Evaluate: $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$.

15. Evaluate: $\int \frac{dx}{3\sin^2 x+4\cos^2 x}$ (OR) $\int \frac{dx}{\sin x+\cos x}$

16. Evaluate: $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}$ dx.

17. Find the points on the curve y = x^3 at which the slope of the tangent is equal to the y-coordinate of the point.

(OR)

Prove that the curves x = y^2 and xy = k cut at right angles if $8k^2$ = 1.

18. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi vertical angle is $\tan^{-1} 0.5$. Water is poured into it at a constant rate of 5 cubic metres per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m. What is the importance of conservation of water?

19. Find inverse of matrix A using column elementary operations where A = $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$.

20. Solve for x: $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$.

21. Prove that $\tan^{-1}\frac{1-x}{1+x}$ - $\tan^{-1}\frac{1-y}{1+y} = \sin^{-1}\frac{y-x}{\sqrt{1+x^2}\sqrt{1+y^2}}$.

(OR)

Prove that: $\tan(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}) + \tan(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}) = \frac{2b}{a}$.

22. If y = $(x\cos x)^x + (\sin x)^{\frac{1}{x}}$, find dy/dx.

23. If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ Show that $A'A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

SECTION - D

24. Evaluate: $\int_{-1}^{4}(x^2 - x)$dx using limit as a sum.

25. Evaluate: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx.$ (OR) Evaluate: $\int \sqrt{\tan x}\, dx.$

26. Show that the height of the cylinder of greatest volume that can be inscribed in a right circular cone of height h and semi vertical angle $\propto$ is one third that of the cone and greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \propto$.

27. Find the sub intervals for which y = $\frac{4\sin\theta}{2+\cos\theta} - \theta$ is increasing or decreasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

28. Using properties of determinants prove that:

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3.$$

(OR)

If p$\neq 0$, q$\neq 0$ and $\begin{vmatrix} p & q & p\propto +q \\ q & r & q\propto +r \\ p\propto +q & q\propto +r & 0 \end{vmatrix}$ = 0 then using the properties of determinants, prove that atleast one of the following statements is true: (i) p, q, r are in G.P (ii) $\propto$ is the root of the equation $px^2+2qx+r=0$.

29. If $(x-a)^2 + (y-b)^2$ = c^2 for some c > 0 . Prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is independent of a and b.

(OR)

If x = asin2t (1+cos2t) and y = bcos2t (1-cos2t), show that $\frac{dy}{dx}$ is $\frac{b\tan t}{a}$.