SECTION – B

Question numbers 11 to 22 carry 4 marks each.

Q.11. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by $a * b = \min\{a, b\}$. Write the operation table of the operation *. *4 marks*

Q.12. Prove the following: *4 marks*

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2},\quad x\in\left(0,\frac{\pi}{4}\right)$$

OR

Find the value of:

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$$

Q.13. Using properties of determinants, prove that: *4 marks*

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Q.14. Find the value of $'a'$ for which the function f defined as: *4 marks*

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0 \\ \dfrac{\tan x-\sin x}{x^3}, & x>0 \end{cases}\quad \text{is continuous at } x=0.$$

Q.15. Differentiate: *4 marks*

$$x^{x\cos x}+\frac{x^2+1}{x^2-1}\ w.r.t.\ x$$

OR

$$If\ x=a(\theta-\sin\theta),\ y=a(1+\cos\theta)\ find\ \frac{d^2y}{dx^2}$$

Q. 16. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. Increasing when the height is 4 cm? *4 marks*

Or

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to $x - axis$.

Q.17. Evaluate: *4 marks*

$$\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx.$$

Or

Evaluate:

$$\int \frac{2x}{(x^2 + 1)(x^2 + 3)} dx$$

Q. 18. Solve the following differential equation: *4 marks*

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$$

Q. 19. Solve the following differential equation: *4 marks*

$$\cos^2 x \frac{dy}{dx} + y = \tan x.$$

Q. 20. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ *4 marks*

Q. 21. Find the angle between the following pair of lines:

$$\frac{-x + 2}{-2} = \frac{y - 1}{7} = \frac{z + 3}{-3} \text{ and}$$

$$\frac{x + 2}{-1} = \frac{2y - 8}{4} = \frac{z - 5}{4}$$

And check whether the lines are parallel or perpendicular. *4 marks*

Q.22. probabilities of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the that (1) the problem is solved (ii) exactly one of them solves the problem. *4 marks*

SECTION-C

Question numbers 23 to 29 carry 6 marks each.

Q.23. Using matrix method, solve the following system of equations: *6 marks*

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2;\quad x,\ y,\ z,\ \neq 0$$

OR

Using elementary transformations, find the inverse of the matrix.

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

Q.24. Show that of all the rectangles inscribed in a given fixed circus, the square has the maximum area. *6 marks*

Q.25. Using integration, find the area of the triangular region whose sides have equations y = 2*x* + 1, y = 3*x* + 1 and *x* = 4. *6 marks*

Q.26. Evaluate: *6 marks*

$$\int_{0}^{\pi/2} 2\sin x\cos x\tan^{-1}(\sin x)\,dx.$$

Q.27. Find the equation of plane which contains the line of intersection of the planes

$\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ **and which is perpendicular to the plane** $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$. *6 marks*

Q.28. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If profit on a racket and on a bat is Rs20 and Rs10 respectively, find the number of tennis racket and crickets bats the factory must manufacture bats that the factory must manufacture to earn the maximum profit. Make it is an L.P.P and solve graphically. *6 marks*

Q.29. Suppose 5% of men and 0.25% of woman have grey hair A grey haired person is selected at random. What is the probability that there are equal number od males and females? *6 marks*