SECTION – B

Question numbers 11 to 22 carry 4 marks each.

Q.11. A family has 2 children. Find the probability that both are boys, if it is known that

(i) at least one of the children is a boy,

(ii) the elder child is a boy. 4 marks

Q.12. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $4\}$ is an equivalence relation. Find the set of all elements related to 1. 4 marks

Q.13. Prove the following: 4 marks

$$\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right).$$

OR

Prove the following:

$$\cos\left[\tan^{-1}\{\sin(\cot^{-1}x)\}\right] = \sqrt{\frac{1+x^2}{2+x^2}}.$$

Q.14. Express the following matrix as the sum of a symmetric and a skew symmetric matrix, and verify your result: 4 marks

$$\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

Q.15. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ find a vector of magnitude 6 unis which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ 4 marks

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 18$.

Q.16. find the points on the line 4 marks

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} \text{ at a distance of } 5 \text{ units from the point } P(1, 3, 3).$$

OR

Find the distance of the point P (6, 5, 9) from the plane determined by the point A (3, -1, 2), B (5, 2, 4) and C (-1, -1, 6).

Q.17. Solve the following differential equation: *4 marks*

$$(x^2-1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1};\ |x| \neq 1$$

OR

Solve the following differential equation:

$$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0.$$

$$\therefore \sqrt{1+y^2} = -\sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + c.$$

Q.18. Show that the differential equation $(x-y)\frac{dy}{dx} = x+2y$, is homogeneous and solve it. *4 marks*

Q. 19. Evaluate the following: *4 marks*

$$\int \frac{x+2}{\sqrt{(x-2)(x-3)}}\,dx.$$

Q.20. Evaluate the following: *4 marks*

$$\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx.$$

Q. 21. $If\ y = e^{a\sin^{-1}x},\ -1 \le x \le 1,\ then\ show\ that$ *4 marks*

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Q. 22. IF *4 marks*

$$y = \cos^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right),\ find\ \frac{dy}{dx}.$$

SECTION – C

Question numbers 23 to 29 carry 6 marks each.

Q. 23. Using properties of determinants, prove the following: *6 marks*

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$$

OR

Find the inverse of the following matrix using elementary operations:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Q.24. A bag contains 7 red, 4 white and 5 black balls. Two balls are drawn at random, from the bag. What is the probability that both the balls are white? *6 marks*

Q.25. One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an L.P.P. and solve it graphically. *6 marks*

Q.26. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P (3, 2, 1) from the plane $2x - y + z + 1 = 0$. Find also, the image of the point in the plane. *6 marks*

Q.27. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$. *6 marks*

OR

Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1) B(6, 6) and C(8, 4).

Q.28. If the length of three sides of a trapezium other than the base is 10 cm each, find the area of the trapezium, when it is maximum. *6 marks*

Q.29. Find the interval in which the following function $f(x) = 20 - 9x^2 + 6x^3$ is

(a) strictly increasing,

(b) strictly decreasing. *6 marks*