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Time: 3 Hours
Total Marks: 100
Note: 1. Attempt all Sections. All the symbols have their usual meaning.

## SECTION A

1. Attempt all questions in brief.

| Qno. | Question | Marks | CO |
| :--- | :--- | :--- | :--- |
| a. | What is the cardinality of the set? <br> Find the cardinality of the set $\{1,\{2, \phi,\{\phi\}\},\{\phi\}\}$. | 2 | 1 |
| b. | Let the two following functions be defined on set of real numbers be as: <br> $\mathrm{f}(\mathrm{x})=2 \mathrm{x}+3$ and $\mathrm{g}(\mathrm{x})=\mathrm{x}^{2}+1$. Find the (fog)(x). | 2 | 1 |
| c. | Define the well-ordered set? Give an example of well-ordered set. | 2 | 2 |
| d. | Draw the Hasse diagram of the lattice of $(\mathrm{D} 6, \mid)$. | 2 | 2 |
| e. | Define Tautology and Contradiction. | 2 | 3 |
| f. | Discuss the truth table of $p \leftrightarrow q$. | 2 | 3 |
| g. | What is the generator of a cyclic group? | 2 | 4 |
| h. | Find the order of each element in the group $(\{1,-1\},).$. | 2 | 4 |
| i. | Find the number of handshakes in party of 12 people, where each two of <br> them shake hands with each other. | 2 | 5 |
| j. | Discuss the pigeonhole principle? | 2 | 5 |

## SECTION B

## 2. Attempt any three of the following:

| Qno. | Question | Marks | CO |
| :--- | :--- | :--- | :--- |
| a. | Prove that the relation $(\mathrm{x}, \mathrm{y}) \in \mathrm{R}$, if $\mathrm{x} \geq \mathrm{y}$ defined on the set of positive <br> integers is a partial order relation. | 10 | 1 |
| b. | If $\mathrm{B}=\{1,3,5,15\}$, then show that $\left(B,+, .,,^{\prime}\right)$ is a Boolean Algebra, <br> where $\mathrm{a}+\mathrm{b}=\mathrm{cm}(\mathrm{a}, \mathrm{b}), \mathrm{a} \cdot \mathrm{b}=\mathrm{gcd}(\mathrm{a}, \mathrm{b})$ and $a^{\prime}=\frac{15}{a}$. | 10 | 2 |
| c. | (i) Prove that conditional proposition and its contrapositive are <br> equivalent, i.e. $(p \rightarrow q) \equiv \sim q \rightarrow \sim p$. <br> (ii) Prove the equivalence: $(p \rightarrow q) \rightarrow q \equiv p \vee q$ | 10 | 3 |
| d. | Show that set $\mathbb{Z}_{6}=\{0,1,2,3,4,5\}$ forms a group with respect to addition <br> modulo 6. | 10 | 4 |
| e. | (i) State all PEANO's axioms. <br> (ii)In how many ways, can 7 boys and 5 girls be seated in a row, so that <br> no two girls may sit together? | 10 | 5 |

## SECTION C

## 3. Attempt any one part of the following:

| Qno. | Question | Marks | CO |
| :--- | :--- | :--- | :--- |
| a. | In a survey of 60 people, it was found that 25 eatApple, 26 eatOrange <br> and 26 eatBanana fruit. Also 9 eat both Apple and Banana, 11 eat both | 10 | 1 |
| Orangeand Apple, and 8 eat both Orangeand Banana. 8eat no fruit at <br> all. Then determine <br> i. the number of people who eat all three fruit. <br> ii. the number of people who eat exactly two fruit. |  |  |  |

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|  | iii. the number of people who eat exactly one fruit |  |  |
| :--- | :--- | :--- | :--- |
| b. | State and Prove De Morgan's laws for set theory. | 10 | 1 |

4. Attempt any one part of the following:

| Qno. | Question | Marks | CO |
| :--- | :--- | :--- | :--- |
| a. | (i)Write the definition of the maximal, minimal, greatest and least <br> element of a Poset. <br> (ii)If $S=\{10,11,12\}$. Determine the power set of S. Draw the Hasse <br> diagram of Poset $(\mathrm{P}(\mathrm{S}), \subseteq)$. <br> (iii)Find the maximal, minimal, greatest and least element of the Poset in <br> Part (ii). | 10 | 2 |
| b. | i) Determine the DNF of Boolean expression $f(x, y, z)=x+y^{\prime} . z$ <br> ii) Simplify the following Boolean expression using K-Map method: <br> $A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B^{\prime} C+A^{\prime} B C+A^{\prime} B C^{\prime}+A B^{\prime} C+A B C$. | 10 | 2 |

5. Attempt any one part of the following:

| Qno. | Question |  |  |
| :--- | :--- | :--- | :--- |
| a. | (i) Given the value of $p \rightarrow q$ is false, determine the value of <br> $(\sim p \vee \sim q) \rightarrow q$. <br> $(i i)$ Prove the equivalence: $(p \rightarrow q) \rightarrow q \equiv p$ | Marks | CO |
| b. | State and Prove De Morgan's laws for propositions using truth table. | $100^{\circ}$ | 3 |

6. Attempt any one part of the following:

| Qno. | Question | Marks | CO |
| :--- | :--- | :--- | :--- |
| a. | Show that set of all integers $\mathbb{Z}$ forms a group with respect to binary <br> operation * defined as $\mathrm{a} * \mathrm{~b}=\mathrm{a}+\mathrm{b}+1$, where $\mathrm{a}, \mathrm{b} \in \mathbb{Z}$. | 10 | 4 |
| b. | (i)Define Ring and Field. Give an example of a Ring and a Field. <br> (ii)Prove that every cyclic group is abelian. | 10 | 4 |

7. Attempt any one part of the following:

| Qno. | Question |  |  |
| :--- | :--- | :--- | :--- |
| a. | State Mathematical Induction. Using the Mathematical Induction, show <br> that $1^{2}+2^{2}+3^{2}+\ldots+n^{2}=\frac{n(n+1)(2 n+1)}{6}, n \geq 1$. | 10 | Marks |
| CO |  |  |  |
| b. | Use generating functions to solve the recurrence relation, <br> $a_{n}-9 a_{n-1}+20 a_{n-2}=0$ where $a_{o}=-3$ and $a_{1}=-10$. | 10 | 5 |

