Printed Pages– 4

CR-2501

M. A./M.Sc. (First Semester) Examination, Nov.-Dec. 2018

MATHEMATICS

Paper : First

(Advanced Abstract Algebra-I)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt quesitons of all two sections as directed. All symbols have their usual meaning.*

Section-A

(Short Answer Type Questions) **5×3=15**

Note : *This section will contain five questions. Internal choice have been provided in each question. Each question will carry 3 marks.*

1. Define normal series, composition series and give one example in each case.

Or

If G is a group and $g \in G$. Then prove that $Tg : G \to G$ defined by $xTg = g^{-1}xg$ is an automorphism.

2. Prove that the commutator subgroup of a group G is a normal subgroup of G.

Or

Show that every Abelian group is solvable.

3. Prove that every finite extension is an algebraic extension.

Or

State and prove factor theorem.

4. Define a prime field and give one example of it.

Or

Prove that a field F is algebraically closed if and only if every irreducible polynomial in $F(x)$ is of degree 1.

5. Define a fixed field and normal extension. Give one example in each case.

Or

Prove that the fixed field of a group of all automorphism G of a field K is a subfield of K.

Section-B

(Long Answer Type Questions) 5×5=25

Note : *This section will contain five questions. Internal choice have been provided in each question. Each question will carry 5 marks.*

6. Every finite group has a composition series prove it.

Or

State and prove Zassenhaus lemma.

7. Prove that a group G is solvable iff $G^{(n)} = e$ for some integer $n \geq 0$.

Or

Show that every nilpotent group is solvable.

8. Prove that an element $a \in K$ is algebraic over F iff $F(a)$ is finite extension of F.

Or

State and prove fundamental theorem of algebra.

9. Show that for any field K the following conditions are equivalent :

(i) K is algebraically closed

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(ii) Every irreducible polynomial in $K(x)$ is of degree 1

(iii) Every non-constant polynomial in $K(x)$ splits in $K(x)$.

Or

Let K and K' be algebraic closures of a field F. Prove that $K \cong K'$ under an isomorphism that is an identity of F.

10. State and prove fundamental theorem of Galois.

Or

If $\sigma_1, \sigma_2, \ldots, \sigma_n$ are distinct auto-morphism of K, then show that it is impossible to find elements $a_1, a_2, \ldots, a_n$ not all zero in K such that

$$a_1\sigma_1(u) + a_2\sigma_2(u) + \ldots + a_n\sigma_n(u) = 0$$

for all $u \in K$.

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