

SE Sem-III (CBSE) - Mechanical - AM - III  
 Applied Mathematics - II  
 Q.P. Code : 538301      25/11/16

( 3 Hours )

[Total Marks : 80]

N.B. (1) Question No.1 is compulsory.

- (2) Attempt any three questions out of the remaining five questions.  
 (3) Figures to right indicate full marks.

- 1 a) Evaluate  $\int_C (\bar{z} + 2z) dz$  along the circle  $x^2 + y^2 = 1$  (5)
- b) Evaluate the integral using Laplace Transform  $\int_0^\infty e^{-t} (t \sqrt{1 + \sin t}) dt$  (5)
- c) Determine the analytic function whose real part is  $u = -r^3 \sin 3\theta$ . (5)
- d) A rod of length  $l$  has its ends A and B kept at  $0^\circ C$  and  $100^\circ C$  respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to  $0^\circ C$  and kept so while that of A is maintained. Find the temperature  $u(x,t)$  at a distance from A and at time  $t$ . (5)
- 2 a) Find complex form of Fourier series of  $f(x) = e^{2x}$  in  $(0,2)$  (6)
- b) Find the orthogonal trajectory of the family of curves given by  $2x - x^3 + 3xy^2 = a$  (6)
- c) Using Bender Schmidt method solve  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$  subject to the conditions  $u(0,t) = 0$ ,  $u(1,t) = 0$ ,  $u(x,0) = \sin \pi x$ ,  $0 \leq x \leq 1$ . Assume  $h=0.2$  (8)
- 3 a) Find  $k$  such that  $\frac{i}{2} \log(x^2 + y^2) + i \tan^{-1}\left(\frac{kx}{y}\right)$  is analytic (6)
- b) Evaluate  $\int_C \frac{1}{(z^3 - 1)^2} dz$  where C is the circle  $|z - 1| = 1$  (6)
- c) Show that the set of functions  $\left\{ \sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \dots \right\}$  forms an orthogonal set over the interval  $[0, L]$ . Construct corresponding orthonormal set. (8)

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TURN OVER

- 4 a) Find Laplace Transform of the periodic function (6)

$$f(t) = \begin{cases} \sin 2t, & 0 < t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \leq t < \pi \end{cases} \quad f(t) = (t + \pi)$$

- b) Find half range sine series for  $x \sin x$  in  $(0, \pi)$  (6)

- c) Expand  $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$  around  $z = 1$  (8)

- 5 a) Using residue theorem evaluate  $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$  where C is  $|z| = 4$  (6)

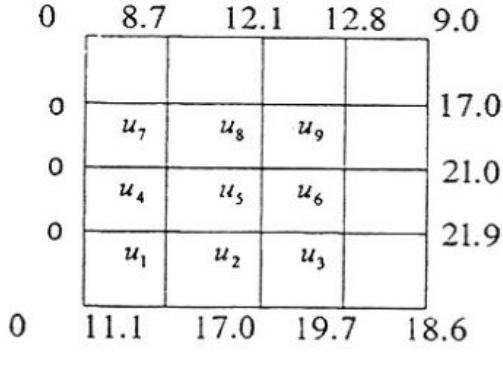
- b) Find Fourier expansion of  $f(x) = x + x^2$  in  $(-\pi, \pi)$  and (6)  
 $f(x+2\pi) = f(x)$

- c) Find i)  $L(e^{-4t} \int_0^t u \sin 3u du)$  ii)  $L^{-1}\left(\frac{1}{s} \log\left(1 + \frac{1}{s^2}\right)\right)$  (8)

- 6 a) Show that the function  $w = \frac{4}{z}$  transform the straight lines  $x = c$  in the z-plane into circles in the w-plane. (6)

- b) Solve using Laplace Transform     $R \frac{dQ}{dt} + \frac{Q}{C} = V$  ,    $Q = 0$  when  $t = 0$       (6)

- c) Solve the Laplace equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for the following data by successive iterations (Calculate first two iterations) (8)



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