S.E. sem III (CBSGS) chemical - AM III 02/12/16
Applied Mathematics-III

Q.P. Code : 536600

(3 Hours)   [Total Marks : 80]

N.B. : (1) Questions **No.1** is **compulsory**.
    (2) Attempt **any three** questions out of the remaining **five** questions.
    (3) **Figures** to the **right** indicate **full marks**.

1. (a) Find the Laplace Transform of $\frac{\cos \sqrt{t}}{\sqrt{t}}$.   5

  (b) Verify Cayley-Hamilton theorem for the matrix A & hence find A^{-1} 5
where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

  (c) Find k such that $\frac{1}{2} \log(x^2 + y^2) + \tan^{-1} \frac{kx}{y}$ is analytic.   5

  (d) The probability that a man aged 60 will live up to 70 is 0.65. What is   5
the probability that out of 10 such men now at 60 at least 7 will live
up to 70?

2. (a) Find inverse Laplace Transform by using Convolution theorem   6
$\frac{1}{s^2 (s+a)^2}$.

  (b) If $f(z) = u + iv$ is analytic & $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. Find $f(z)$.   6

  (c) Find a matrix P that diagonalises the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ and   8

determine $P^{-1} A P$.

3. (a) Find inverse Laplace Transform of $\frac{\left(s^2 + 2s + 3\right)}{\left(s^2 + 2s + 5\right)\left(s^2 + 2s + 2\right)}$.   6

  (b) Find the Bilinear transformation which maps the points $z = -1, 1,$ 6
∞ onto the points $w = -i, -1, i$.

  (c) Using the Kunh - Tucker conditions solve the following N.L.P.P.   8
Maximise : $z = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$
Subject to : $x_1 + x_2 \leq 2$
$2x_1 + 3x_2 \leq 12$
$x_1, x_2, x_3 \geq 0$

TURN OVER

750-1

4.

(a) Evaluate $\displaystyle\int_{1-i}^{2+i} (2x+iy+1)\, dz$ along the curve $x = t+1$ & $y = 2t^2 - 1$.   **6**

(b) Calculate the coefficient of correlation from the following data :   **6**

x	62	64	65	69	70	71	72	74
y	126	125	139	145	165	152	180	208

(c) Reduce the following quadratic form   **8**

$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2\,x_3 - 4x_3\,x_1 + 12x_1\,x_2$ to normal form through congruent transformation. Also find its rank, signature & value class.

5.

(a) Evaluate $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta$.   **6**

(b) The marks obtained by the students in a college are normally distributed with mean 65 & variance 25. If three students are selected at random from this college what is the probability that at least one of them would have scored more than 75 marks?   **6**

(c) Find the orthogonal matrix which diagonalises the real symmetric   **8**

matrix where $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$.

6.

(a) Using Lagrangian multiplier method solve the following N.L.P.P.   **6**

Optimise : $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to : $x_1 + x_2 = 4,\ x_1, x_2 \geq 0$

(b) Find the eigen values & eigen vectors for $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$.   **6**

(c) Find inverse Laplace Transform of   **8**

(i) $\dfrac{e^{4-3s}}{(s+4)^{5/2}}$

(ii) $\tan^{-1}\left(\dfrac{s+a}{b}\right)$.

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150-2